

12. अर्धव्यास क्रमशः 4 से.मि., 5 से.मि. र 5 से.मि. भएका तिन ओटा गोलालाई पगालेर एउटै गोला बनाउँदा उक्त गोलाको आयतन कति होला ? पत्ता लगाउनुहोस् ।

Three solid spheres of iron whose diameters are 4 cm, 5 cm and 5 cm respectively are melted into a single solid sphere. Find the volume of the solid sphere.

Here, let the radius of the solid sphere be R cm. Then,

We have,

Volume of the solid sphere = Sum of the volume of three spheres of radii 4 cm, 5 cm and 5 cm respectively.

$$\text{So, } \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(4)^3 + \frac{4}{3}\pi(5)^3 + \frac{4}{3}\pi(5)^3$$

Dividing both the sides by $\frac{4}{3}\pi$

$$\text{or, } R^3 = 4^3 + 5^3 + 5^3 \quad \text{or, } R^3 = 314$$

$$\text{Now, Volume of new sphere} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times \frac{22}{7} \times 314 = 1315.81 \text{ cm}^3$$

Thus, the volume is 1315.81 cm^3 .

6.3 संयुक्त ठोस (COMBINED SOLID)

1. तल दिइएका ठोस वस्तुको सतहको क्षेत्रफल र आयतन निकाल्नुहोस् :

Find the surface area and volume of following solid objects:

(a) Radius (r) = 7 cm and height (h) = 21 cm

We have,

$$\begin{aligned} \text{Surface area} &= 3\pi r^2 + 2\pi rh = \pi r(3r + 2h) = \frac{22}{7} \times 7(3 \times 7 + 2 \times 21) \\ &= 22 \times 63 = 1386 \text{ cm}^2 \end{aligned}$$

$$\text{Now, Volume} = \frac{2}{3}\pi r^3 + \pi r^2 h = \pi r^2 \left(\frac{2}{3}r + h \right) = \frac{22}{7} \times 7^2 \left(\frac{2}{3} \times 7 + 21 \right) = 3952.67 \text{ cm}^3$$

Thus, the surface area and volume are 1386 cm^2 and 3952.67 cm^3 .

(b) Radius = 3.5 cm and height of cylinder (h) = 35 cm

We know that,

$$\begin{aligned} \text{Surface area} &= 4\pi r^2 + 2\pi rh = 2\pi r(2r + h) \\ &= 2 \times \frac{22}{7} \times 3.5(2 \times 3.5 + 35) = 924 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Again, Volume (V)} &= \frac{4}{3}\pi r^3 + \pi r^2 h = \pi r^2 \left(\frac{4}{3}r + h \right) = \frac{22}{7} \times (3.5)^2 \left(\frac{4}{3} \times 3.5 + 35 \right) \\ &= \frac{77}{2} \times \frac{119}{3} = 1527.17 \text{ cm}^3 \end{aligned}$$

Thus, the surface area and volume are 924 cm^2 and 1527.17 cm^3 respectively.

(c) Here, height of cylinder = 28 cm - 3.5 cm \therefore h = 24.5 cm and Radius (r) = 3.5 cm

We know that,

$$\begin{aligned} \text{Surface area} &= 3\pi r^2 + 2\pi rh = \pi r(3r + 2h) = \frac{22}{7} \times 3.5(3 \times 3.5 + 2 \times 24.5) \\ &= 616 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Again, Volume} &= \frac{2}{3}\pi r^3 + \pi r^2 h = \pi r^2 \left(\frac{2}{3}r + h \right) = \frac{22}{7} \times (3.5)^2 \left(\frac{2}{3} \times 3.5 + 24.5 \right) \\ &= 1033.08 \text{ cm}^3 \end{aligned}$$

Thus, the surface area and volume are 616 cm^2 and 1033.08 cm^3 respectively.

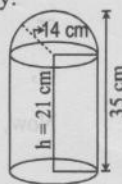
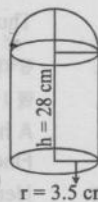
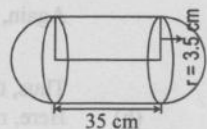
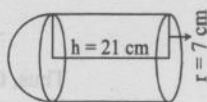
(d) Here height (h) = 21 cm, radius (r) = 14 cm

We know that,

$$\begin{aligned} \text{Surface area} &= 3\pi r^2 + 2\pi rh = \pi r(3r + 2h) = \frac{22}{7} \times 14(3 \times 14 + 2 \times 21) \\ &= 44 \times 84 = 3696 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Again, Volume} &= \frac{2}{3}\pi r^3 + \pi r^2 h = \pi r^2 \left(\frac{2}{3}r + h \right) = \frac{22}{7} \times 14^2 \left(\frac{2}{3} \times 14 + 21 \right) \\ &= 18685.33 \text{ cm}^3 \end{aligned}$$

Thus, the surface area and volume are 3696 cm^2 and 18685.33 cm^3 .

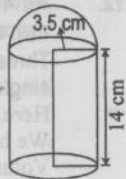


- (e) Here radius (r) = 3.5 cm, height (h) = 14 cm

We know that,

$$\begin{aligned}\text{Surface area} &= 3\pi r^2 + 2\pi r h = \pi r (3r + 2h) \\ &= \frac{22}{7} \times 3.5(3 \times 3.5 + 2 \times 14) = \frac{847}{2} = 423.50 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Again, Volume} &= \frac{2}{3}\pi r^3 + \pi r^2 h = \pi r^2 \left(\frac{2}{3}r + h\right) \\ &= \frac{22}{7} \times (3.5)^2 \left(\frac{2}{3} \times 3.5 + 14\right) = \frac{3773}{6} = 628.83 \text{ cm}^3\end{aligned}$$



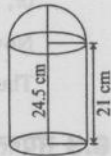
Thus, the surface area and volume are 423.50 cm² and 628.83 cm³ respectively.

- (f) Here, Radius (r) = (24.5 - 21) cm = 3.5 cm, Height of cylinder (h) = 21 cm

We know that,

$$\begin{aligned}\text{Surface area} &= 3\pi r^2 + 2\pi r h = \pi r (3r + 2h) \\ &= \frac{22}{7} \times 3.5(3 \times 3.5 + 2 \times 21) = \frac{1155}{2} = 577.50 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Again, Volume} &= \pi r^2 h + \frac{2}{3}\pi r^3 = \pi r^2 \left(h + \frac{2}{3}r\right) \\ &= \frac{22}{7} \times 3.5 \left(21 + \frac{2}{3} \times 3.5\right) = \frac{770}{3} = 256.67 \text{ cm}^3\end{aligned}$$



Thus, the surface area and volume are 577.50 cm² and 256.67 cm³ respectively.

- (g) Here, radius (r) = 6 cm

Height of cylinder (h) = (42 - 2 × 6) cm = 30 cm

We know that,

$$\begin{aligned}\text{Surface area} &= 4\pi r^2 + 2\pi r h = 2\pi r(2r + h) = 2 \times \frac{22}{7} \times 6(2 \times 6 + 30) \\ &= 1584 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Again, Volume (V)} &= \frac{4}{3}\pi r^3 + \pi r^2 h = \pi r^2 \left(\frac{4}{3}r + h\right) = \frac{22}{7} \times 6^2 \left(\frac{4}{3} \times 6 + 30\right) \\ &= 4299.43 \text{ cm}^3\end{aligned}$$

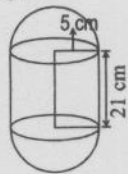
Thus, the surface area and volume are 1584 cm² and 4299.43 cm³ respectively.

- (h) Here, radius (r) = 5 cm and Height of cylinder (h) = 21 cm

We know that,

$$\text{Surface area} = 4\pi r^2 + 2\pi r h = 2\pi r(2r + h) = 2 \times \frac{22}{7} \times 5(2 \times 5 + 21) = 974.28 \text{ cm}^2$$

$$\begin{aligned}\text{Again, Volume} &= \pi r^2 h + \frac{4}{3}\pi r^3 = \pi r^2 \left(h + \frac{4}{3}r\right) = \frac{22}{7} \times 5^2 \left(21 + \frac{4}{3} \times 5\right) \\ &= 2173.81 \text{ cm}^3\end{aligned}$$



Thus, the surface area and volume of the solid object are 974.28 cm² and 2173.81 cm³ respectively.

2. उचाइ 42 से.मि. भएको बेलनामाथि एउटा अर्धगोला राखिएको छ, जसको वक्र सतहको क्षेत्रफल 308 वर्ग से.मि. छ। उक्त संयुक्त ठोस वस्तुको आयतन निकाल्नुहोस्।

A hemisphere whose curved surface area 308 cm² is placed over a cylinder having height 42 cm. Find the volume of the solid.

Here, height of cylinder (h) = 42 cm

We have given, CSA (hemisphere) = 308

$$\text{or, } 2\pi r^2 = 308$$

$$\text{or, } 2 \times \frac{22}{7} \times r^2 = 308$$

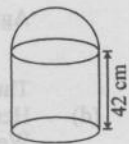
$$\text{or, } r^2 = 49$$

$$\therefore r = 7 \text{ cm}$$

$$\text{Now, volume of the solid (V)} = \frac{2}{3}\pi r^3 + \pi r^2 h = \pi r^2 \left(\frac{2}{3}r + h\right)$$

$$= \frac{22}{7} \times 7^2 \left(\frac{2}{3} \times 7 + 42\right) = \frac{21560}{3} = 7186.67 \text{ cm}^3$$

Thus, the volume of the solid is 7186.67 cm³.



3. आधारको क्षेत्रफल 616 वर्ग से.मि. भएको बेलनामाथि अर्धगोला राखिएको छ । यदि बेलनाको उचाइ 35 से.मि. भए उक्त संयुक्त ठोस वस्तुको पुरा सतहको क्षेत्रफल निकाल्नुहोस् ।

A hemisphere is placed over a cylinder whose volume is 616 sq cm. If the height of the cylinder is 35 cm, find the total surface area of the solid object.

Here, Area of base = 616 cm²

$$\text{or, } \pi r^2 = 616 \quad \text{or, } \frac{22}{7} \times r^2 = 616$$

$$\text{or, } r^2 = 196 \quad \therefore r = 14 \text{ cm}$$

$$\begin{aligned} \text{Now, total surface area} &= 3\pi r^2 + 2\pi rh = \pi(3r + 2h) \\ &= \frac{22}{7} \times 14(3 \times 14 + 2 \times 35) = 4928 \text{ cm}^2 \end{aligned}$$



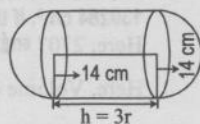
Thus, the total surface area of the solid is 4928 cm².

4. अर्धव्यासको 3 गुना उचाइ भएको बेलनाको दुवैतिर अर्धगोला राखिएका छन् यदि अर्धगोलाको अर्धव्यास 14 से.मि. भए उक्त संयुक्त ठोस वस्तुको आयतन निकाल्नुहोस् ।

Two hemispheres whose radius 14 cm are placed at the both ends of a cylinder whose height is 3 times the radius. Find the volume of the solid object.

Here, height of cylinder (h) = 3r = 3 × 14 = 42 cm

$$\begin{aligned} \text{We know that, Volume (V)} &= \frac{4}{3}\pi r^3 + \pi r^2 h = \pi r^2 \left(\frac{4}{3}r + h \right) \\ &= \frac{22}{7} \times 14^2 \left(\frac{4}{3} \times 14 + 42 \right) = \frac{121112}{3} = 37370.67 \text{ cm}^3 \end{aligned}$$



Thus, the volume of the solid is 37370.67 cm³.

5. आयताकार खेल मैदानको चौडाइको दुवैतिर अर्धवृत्ताकार पोखरीहरू छन्, यदि खेल मैदानको चोडाइ 21 मिटर र पोखरीको गहिराइ 1.5 मिटर भए पोखरीको दुवैतिर कति पानी अटाउँछ ? पत्ता लगाउनुहोस् ।

Two semi circular ponds are on both the sides of a rectangular play ground. If breadth of the playground is 21 m and depth of the pond 1.5 m, how many litres of water can the ponds hold ? Find it.

Here, diameter of semi circle (d) = 21 m

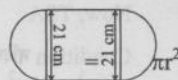
We know that, Area of two semi circles

$$= \pi \left(\frac{d}{2} \right)^2 = \frac{22}{7} \left(\frac{21}{2} \right)^2 = \frac{693}{2} = 346.50 \text{ m}^2$$

Again, Volume of the ponds = Area of ponds × depth = 346.50 m² × 1.5 m

$$= 519.75 \text{ m}^3 = 519.75 \times 1000 \ell = 51975000 \ell$$

Thus the ponds hold 51975000 ℓ of water.



6. एउटा बेलनाकार ट्याङ्की माथिको भाग अर्ध गोलाकार छ । उक्त ट्याङ्कीको कुल उचाइ 4.5 मिटर र आधारको परिधि 22 मिटर छ भने ट्याङ्कीमा कति लिटर पानी अटाउँछ ? पत्ता लगाउनुहोस् ।

The upper part of a cylindrical tank is hemispherical. If the height of the tank is 4.5 m and circumference of base 22 m, how many litres of water, the tank can hold ?

Here, circumference of base (c) = 22 m or, 2πr = 22

$$\text{or, } 2 \times \frac{22}{7} \times r = 22 \quad \text{or, } r = \frac{7}{2} = 3.5 \text{ m}$$

$$\text{Now, Volume} = \frac{2}{3}\pi r^3 + \pi r^2 h = \pi r^2 \left(\frac{2}{3}r + h \right) = \frac{22}{7} \times 3.5^2 \left[\frac{2}{3} \times 3.5 + (4.5 - 3.5) \right] = \frac{385}{3}$$

$$\therefore \text{Volume} = 128.33 \text{ cm}^3$$

Thus, the volume of the solid is 128.33 cm³.



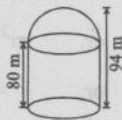
7. एउटा बेलानामाथि अर्धगोला भएको ठोस वस्तुमा बेलनाको अर्धव्यास र अर्धगोलाको अर्धव्यास बराबर छन् । यदि संयुक्त ठोस वस्तुको कुल उचाइ 94 से.मि. र बेलनाको उचाइ 80 से.मि. भए सो ठोस वस्तुको पुरा सतहको क्षेत्रफल र आयतन पत्ता लगाउनुहोस् ।

A hemisphere is on the top of a cylinder in a solid object in which radius of hemisphere and cylinder are equal to each other. If the total height of the combined solid is 94 cm and height of the cylinder is 80 m. Find the total surface area and volume of the solid.

Here, radius (r) = total height - height of cylinder = 94 m - 80 m = 14 m

We know that,

$$\begin{aligned} \text{TSA of the solid} &= 3\pi r^2 + 2\pi rh = \pi r(3r + 2h) \\ &= \frac{22}{7} \times 14(3 \times 14 + 2 \times 80) = 8888 \text{ cm}^2 \end{aligned}$$



$$\begin{aligned} \text{Again, Volume of the solid} &= \frac{2}{3}\pi r^3 + \pi r^2 h = \pi r^2 \left(\frac{2}{3}r + h \right) \\ &= \frac{22}{7} \times 14^2 \left(\frac{2}{3} \times 14 + 80 \right) = \frac{165088}{3} = 55029.33 \text{ cm}^3 \end{aligned}$$

Thus, the TSA and volume of the solid are 8888 cm² and 55029.33 cm³.

8. बेलनाको अर्धव्यास र अर्ध गोलाको अर्धव्यास बराबर भएको एउटा ठोस वस्तु छ, जसको पुरा आयतन 130284 घन से.मि. छ । यदि बेलनाको मात्र उचाइ 80 से.मि. भए उक्त ठोस वस्तुको पुरा सतहको क्षेत्रफल पत्ता लगाउनुहोस् ।

A solid is made by a hemisphere and cylinder having equal radii. The volume of the solid is 130284 cm³. If the height of the cylinder is 80 cm, find the total surface area of the solid.

Here, 2707 लाई 130284 बनाउनुहोला र Condition: $r^3 = 22r^2 - 441$ यन्नुहोला ।]

$$\text{Here, Volume of the solid} = 130284 \text{ cm}^3 \quad \text{or, } \frac{2}{3}\pi r^3 + \pi r^2 h = 130284$$

$$\text{or, } \pi r^2 \left(\frac{2}{3}r + h \right) = 130284$$

$$\text{or, } r^2 \left(\frac{2}{3}r + 80 \right) = 41454$$

$$\text{or, } r^2 (2r + 240) = 124362$$

$$\therefore r^3 + 120r^2 - 62181 = 0$$

[Using condition Condition: $[r^3 = 22r^2 - 441]$ यन्ने]

$$\text{or, } 22r^2 - 441 + 120r^2 - 62181 = 0$$

$$\text{or, } r^2 = 441$$

$$\text{or, } 142r^2 = 62622$$

$$\therefore r = 21$$

$$\text{Now, TSA} = 3\pi r^2 + 2\pi rh = \pi r(3r + 2h) = \frac{22}{7} \times 21(3 \times 21 + 2 \times 80) = 14718 \text{ cm}^2$$

Condition नदिए

$$\text{or, } r^3 - 21r^2 + 141r^2 - 2961r + 2961r - 62181 = 0$$

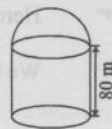
$$\text{or, } r^2(r - 21) + 141r(r - 21) + 2961(r - 21) = 0$$

$$\text{or, } (r - 21)(r^2 + 141r + 2961) = 0$$

$$\text{Either } r - 21 = 0 \quad \therefore r = 21$$

or, $r^2 + 141r + 2961 = 0$ which gives (-)ve values so it is impossible. Since all the terms are positive.

$$\text{Now, TSA} = 3\pi r^2 + 2\pi rh = \pi r(3r + 2h) = \frac{22}{7} \times 21(3 \times 21 + 2 \times 80) = 14718 \text{ cm}^2$$



9. बेलना र अर्धगोलाको संयुक्त उचाइ 10.5 से.मि. तथा बेलनाको आधारको व्यास 7 से.मि. छ । उक्त ठोस वस्तुको पुरा सतहको क्षेत्रफल आयतन पत्ता लगाउनुहोस् ।

The combined height of cylinder and hemisphere is 10.5 cm and diameter of base of cylinder is 7 cm. Find the total surface area and volume of the solid.

Here, radius (r) = $\frac{7}{2}$ cm = 3.5 m and Height of cylinder (h) = (10.5 - 3.5) m = 7 m

We know that,

$$\begin{aligned} \text{Total surface} &= 3\pi r^2 + 2\pi rh = \pi r(3r + 2h) \\ &= \frac{22}{7} \times \frac{7}{2} \left(3 \times \frac{7}{2} + 2 \times 7 \right) = 11 \times 24.5 = 269.50 \text{ cm}^2 \end{aligned}$$



$$\text{Again, Volume} = \frac{2}{3}\pi r^3 + \pi r^2 h = \pi r^2 \left(\frac{2}{3}r + h \right) = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \left(\frac{2}{3} \times \frac{7}{2} + 7 \right) = \frac{1078}{3} = 359.33 \text{ cm}^3$$

Thus, the TSA and volume of the solid are 269.50 cm² and 359.33 cm³ respectively.

10. अर्धगोला र बेलनाको अर्धव्यास बराबर भएको ठोस वस्तुमा अर्धव्यास र बेलनाको उचाइको अनुपात 1:3 तथा तिनीहरूको योगफल 14 से.मि. छ। सो ठोस वस्तुको पुरा सतहको क्षेत्रफल र आयतन पत्ता लगाउनुहोस्।
 In a solid object the radius of hemisphere and cylinder are equal. The ratio of radius and height of a cylinder is 1.3 and their sum 14 cm. Find the total surface area and volume of the solid. ...

Here, $\frac{r}{h} = \frac{1}{3}$

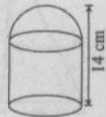
$\therefore h = 3r$

or, $r + h = 14$

or, $r + 3r = 14$

or, $4r = 14$

$\therefore r = \frac{7}{2} = 3.5 \text{ cm}$



and $h = 3r = 3 \times \frac{7}{2} = 10.5 \text{ cm}$

We know that,

$$\text{TSA} = 3\pi r^2 + 2\pi r h = \pi r (3r + 2h) = \frac{22}{7} \times 3.5 (3 \times 3.5 + 2 \times 10.5) = \frac{693}{2} \text{ cm}^2 = 346.50 \text{ cm}^2$$

$$\text{Now, Volume (V)} = \frac{2}{3} \pi r^3 + \pi r^2 h = \pi r^2 \left(\frac{2}{3} r + h \right) = \frac{22}{7} (3.5)^2 \left[\frac{2}{3} \times 3.5 + 10.5 \right] = \frac{5929}{12} = 494.08 \text{ cm}^3$$

Thus, the TSA and volume are 346.50 cm^2 and 494.08 cm^3 .